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A Probabilistic Approach to Fatigue and Failure of Composite Materials

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0.1 Abstract

With more advances being made everyday in material design, it is apparent that a deterministic approach for solving problems is increasingly inadequate. Deterministic analysis relies too much on assumptions that a given material will stay homogenous throughout, as well as the fact that variations in material properties can be neglected. The probabilistic analysis; however, takes into account that not all materials are created equal, even if they are from the same factory. This paper will examine a particular material and show the differences between the two approaches in solving a fatigue based problem.

0.2 Introduction

An Engineer has a duty they must perform to society, and that is to keep the well-being of the public held in the utmost regard. Because of this, if another method is available and it provides more accurate results, it should be used over the more generalized equation in order to get a better look at the problem as a whole.

Prediction of fatigue life in composite components is paramount when being used in an application that directly affects the safety and welfare of the general public, whether it be one person or hundreds of people who might be injured should failure occur. Composite materials are typically made from an epoxy matrix with the actual fibers being any different combination of materials, though this paper will look at carbon fibers in particular. Given that many uncertainties can arise from the composite being formed such as the matrix being distributed unevenly, a difference in thickness of the carbon fibers themselves, the layers being formed with varying thicknesses, and even imperfect bonding between the layers, an engineer must be careful in assumptions made so as not to build up the integrity of the part when it actually posses far less strength than calculated. In previous studies, the standard methods employed for variable fatigue life prediction far overestimate the experimentally measured life. [Rognin et. all, 2009]

A common method employed by engineers when analyzing a material in fatigue is using the Modified-Goodman diagram. This method does a good job of predicting the failure point of a material during fatigue, as it is much lower than one would find when just using a materials yield strength (S_y). The problem with this method is that it does not assume a stochastic material, also the endurance limit (S_e) must be known for this method to work. This need for an endurance limit presents a problem when analyzing a Carbon fiber re-enforced polymer (CFRP) since there is no actual S_e given for this material. This leads us to look at experimental data for composites in order to find a rough estimate in life-in-cycles and finally apply this into the Palmgren-Miner method.

0.3 Analyzing the Traditional Method of Engineering Design

The previously mentioned Palmgren-Miner's Method shows how a typical fatigue problem under a load is analyzed. This method is seen as:

$$D = \sum_{k=1}^{i} \frac{n_i}{N_i} \tag{1}$$

Where the applied cycles (n_i) are divided by life-in-cycles (N_i) to give the amount of life used at a particular stress applied. This equation is utilized for all given applied cycles to give us the cumulative summation of the life used. N_i is derived from experimental data which is found in the material's properties. To ensure that the part does not fail, D must stay less than 1.

This method; however, is not completely sound. An Engineer may find that some percentage of life is used at one point and move on to the next number of applied cycles at a given stress. The next percentage of life solved for could show different results than what is seen experimentally. The material is not guaranteed to behave the same throughout the entire process so some uncertainty must be accounted for.

0.4 Safety and Reliability

A new method that has been emerging over the past few years is the method of reliability. Many companies are slowly incorporating the use of reliability, while some companies such as GE use reliability as their sole method of predicting failure for their products. The reliability method employs the use of statistics rater than simply using a base number such as yield strength or the endurance limit as the entire generalization for a material.

Reliability, when calculated, gives the probability of failure for a given product. This is helpful because it will give the company a base number which is the number of products for a given batch that are expected to fail. This number helps the company set a target probability in order to maximize profits and minimize defects which slow down production time. A company may set a target probability of 10^{-4} for a set of components where one million were produced. This would mean that out of the initial one million components, the company could expect 100 of those to be defective.

0.5 Limits of the Palmgren-Miner's Method

The Palmgren-Miner's Method is good to use as a starting point; however, when looked to for use in structures or parts that have the propensity to endanger human lives should they fail, the method should be improved upon. The life-in-cycles that can be found in any material properties chart is simply taken as canon by many engineers. This value should be taken with a grain of salt because its origins aren't exactly known. The value would change if at anytime during the assembly process a variability of the interaction conditions between bolted parts, adhesive joints, or other types of joints was introduced. [Bogdanovich et. all, 1998]

Another problem is looking to the cycles applied at a given stress level. The variability between cyclic loadings may be minute, so much so that a normal operator would never know, yet the variation could be enough to skew the results. Engineers must come to the realization that as things are built larger and more precise, no longer can values be taken statically and results ran without variation shown.

0.6 Reasons to Consider the Reliability Method

Since composite structures are made of two different materials, uncertainties result in dispersion in the material properties. This makes the reliability method more appealing which allows engineers to model the problem with random loadings and material properties in a single problem. [Onkar et. all, 2005] This is important for everybody starting with the company and trickling all the way down to the consumer. The capability to analyze multiple random variables (RVs) such as the life-in-cycles and the applied cycles from a specified stress helps to produce a much more accurate model. The results given from the probabilistic analysis will help the engineer to provide his company with the most cost-effective model that can still be maintained as safe.

A common method to analyzing structures probabilistically is the weakest-link design. This method shows the failure probability and probabilistic location of failure in the structure or component being analyzed by looking at the structure as a whole and singling out the weakest member where failure is most likely to occur. [Onkar et. all, 2005]

A very compelling reason for a company to consider using the reliability method is the sheer amount of savings from being able to plan for defects. The previously mentioned probability of failure (section 0.4) provides companies with invaluable information. The company may look at a batch of product being produced, whether it be vehicles or beams, and specify that their target probability is 10^{-4}. With this probability set, from the million products produced in the batch the company can now plan for 100 of them to come back as defective. When the company can plan for defects, turn-around time is much faster for both the company and the consumer.

One problem hindering adoption of the reliability method is the lack of understanding by both management and engineers. Many engineers are set in their ways and do not want to learn anything but the deterministic way they have come to know and love. Management may see this as something that requires even more training for their employees and they fail to see just how useful it would be to their company so they are quick to dismiss it. Until this method receives mainstream exposure, it will still be under-utilized by most engineers. Another reason is the experimental data that is sometimes required for these problems. Obtaining the experimental data is a costly and time-consuming operation that some businesses just do not have the means to provide.

0.7 Uses of Probabilistic Design

Probabilistic design can be used in a multitude of applications and not just limited to failure of a product. Composites are widely used in naval, aerospace, automotive, civil engineering, and many other fields where they are exposed to cyclic loading [Liu et. all, 2005]. Nearly any type of data that can be shown with a distribution can have an application with the Reliability method. Whether it is NASA analyzing a comet and potential impact of the Earth, a local water municipal analyzing the amount of water that will arrive to a location through a pipeline, or how much lift a wing design on a wind turbine will produce. Probabilistic design can also be extended to current products without a distribution if various assumptions are made. In many design problems, one can assume that material properties can vary by 10 percent and create a distribution off of those values alone. These of course will not be near as reliable, but it is a proof of concept showing just how versatile this method truly is.

0.8 The Future of Probabilistic Design

Probabilistic design has faced rejection for many years, but has gained momentum recently. Companies realize the need for damage control and that no matter how well a company is run there will always be defects. Smart companies realize this right now and are the ones who currently use the reliability method while others have some catching up to do. The fact that Civil Engineers are all required to have and demonstrate some knowledge on the subject [Haldar and Mahadevan xiii] should be indicative that mechanical engineering students will soon follow their path.

0.9 Reliability Method Application

Composite materials have taken the mountain bike and road bicycle world by storm in the last five years. In the early 2000s, a frame would typically be made of 6061 T6 series Aluminum with some 7005 series making their way into the mix. As the price on composite materials, especially CFRP, began to drop more and more, companies switched their attention to this material with an extremely high strength to weight ratio. The yield strength of CFRP is very high so it has become a point of less focus while the fatigue strength of the frame has moved into the light. Since CFRP has no specified endurance limit, a probabilistic analysis to fatigue is almost the only way to get a valid number for fatigue strength.

To begin, some assumptions must be made in order for a stress to be found which will give us the applied cycles at a given stress. An interesting point to note about the failure of carbon frames is that instead of failing at the headtube like a normal 6061 Al frame would, many carbon frames fail at the top tube due to the fact that there is no weld to fail at and also that CFRP is much stronger in tension as opposed to compression. This information has us focus our possible failure point due to fatigue at the top tube. An average downhill mountain biker can be assumed to have a weight of 180 pounds. The top tube inner diameter (d_i) and outer diameter (d_o) are set to 0.1575 inches and 1.5 inches respectively. The area of the top tube can be found from:

$$A = \frac{\pi(d_o^2 - d_i^2)}{4} \tag{2}$$

Evaluating Eqn. (2) at the given diameters, the area is found to be $2.67e^{-4}$ m^2 when converted to SI units. The max stress to be specified comes from the 180 pound rider jumping off of a cliff at a height of 6.5 feet and landing at an angle of 40°. The normal equation F=ma equation will not work for this problem, instead another equation must be formulated [O'Shea, 2004]:

$$F_n = mg(1 + \frac{h}{b})cos\theta \tag{3}$$

Where m is the mass of the rider, g is the force of gravity, h is the height of the drop, b is the distance the rider's center of gravity displaces from a neutral position upon landing (b=1 foot), and θ is the landing angle.

Plugging all of our knowns into Eqn. (3) and converting to SI units, we find that the max force the top tube will experience on a normal ride to be 5749.32 Newtons. Now this is used to find the stress (σ)

$$\sigma = \frac{F_n}{A} \tag{4}$$

The max stress applied to the top tube under normal downhill conditions is found to be 21.533 Mpa. This stress value is used in the S-N diagram [Kawai et. all, 2001].

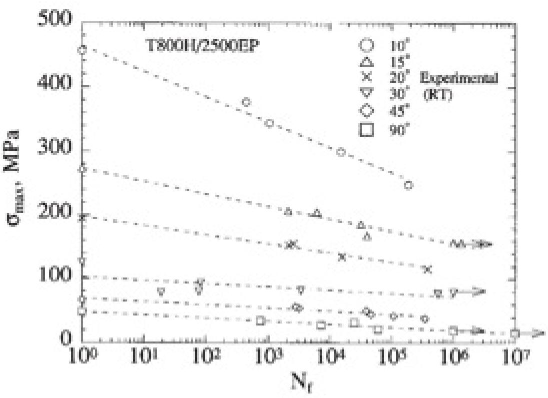

FIGURE 1: S-N Diagram for T800H/epoxy matrix composite.

Using Figure 1, the life in cycles is found to be 10^6. The other assumption to be made is the cycles applied. A typical rider will own this frame for 5 years, ride 3 times per week, and cycle through this stress 100 times per ride to give an applied cycle number of 78,000 cycles. Applying these two values into Eqn. (1) the total life used is found to be D=0.078. This number is far from approaching the total life of the part so it is deemed extremely safe deterministically.

Approaching this problem probabilistically; however, takes into account variations in material properties that is not seen in the previous results. The percent of life used in the reliability case accounts for variations where D can be represented as

$$D = D(n, f, T) \tag{5}$$

Where D is a function of: applied cycles (n), frequency of loading (f), and time (T) [Kamiski, 2002]. Using a normal distribution and a covariance of 10 percent, the distributions are found to be

$$n_i = N(78,000, 7800) \tag{6}$$

$$N_i = N(1,000,000, 100,000) \tag{7}$$

From Miner's method, the limit state for failure is said to be 1 which is when applied cycles are equal to life in cycles. With the limit state specified, the performance function is

$$g() = 1 - \sum_{k=1}^{i} \frac{n_i}{N_i} \tag{8}$$

Using the deterministic method for the downhill mountain bicycle frame, a probabilistic engineer would find that the safety index (β) equal to 9.192 and the probability of failure (p_f) equal to $1.9328e^{-18}$. A target probability of failure for a typical mountain bicycle frame is $10^{-}6$ as found through correspondence to Specialized Bicycle Components. The found probability of failure is much less than the target so it is found that this frame design works under the assumptions given. This part, when analyzed probabilistically, is extremely safe. These results; however, are not entirely true as many assumptions were made in order to simplify the problem. In actuality, there is no landing that will always be a perfect 40° nor will every jump height be 6.5 feet. Some riders have pushed the bar past 30 feet and as the sport progresses so will the heights that people take to with these frames.

The sensitivity levels (Figure 2) which were plotted from NESSUS show just how much assumptions can affect the probability of failure. Here the standard deviation for oi (Life-in-cycles) is the biggest factor to change the p_f. The deviation from N_i is 100,000

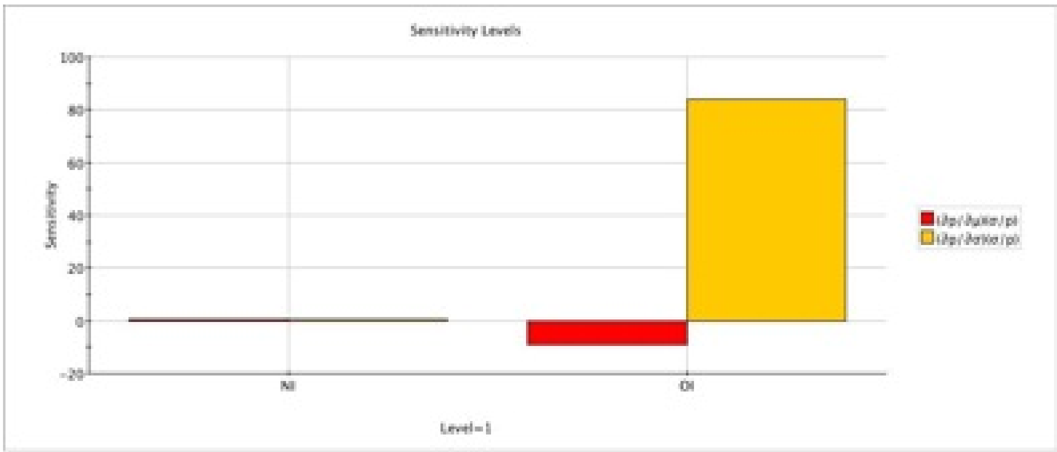

FIGURE 2: Sensitivity Levels

which explains why it has such an effect on this problem. Another reason that the safety index was so high for this part is that static loading was not taken into consideration for this problem. This would be equal to just the force of the rider sitting on the bike while on a flat surface. This is hard to account for because on a downhill mountain bike trail there are almost no flat spots where this could be taken into account. This would also give a very small stress which would not even register on Figure 1 to find life in cycles for it to be included in the summation. If the life-in-cycles is wished to be increased, the largest factors to keep in mind would be the area of the top tube (Eqn. 2) and also the rider's weight. Both of these methods prove that the design is safe under the many assumptions made, but if variations were refined even more, the probabilistic method would prove itself to be the most reliable of the two.

0.10 Conclusion

When two different methods to solve a problem are presented, there is no right or wrong method to pick, but one can always be seen as "more correct". Either the deterministic or probabilistic methods could be used to solve a given problem, and both will produce an answer, but it is up to the engineer to look at all circumstances surrounding the problem to decide the risks involved with the method he or she picks. Too much faith may be put into a deterministic approach because that may be all that the engineer has been taught and this leads to a slippery slope. If the engineer knows that the probabilistic method is the better choice yet has not been taught how to properly utilize it, the results could be just as disastrous as doing it deterministically without any calculations.

Engineers should see the need to learn the probabilistic method as the time spent analyzing why something failed could be dramatically decreased. Failure cases take a large portion of time to solve and implementing a process to account for these failures would not only save engineers time and companies money, consumers would also reap the benefits of this as well.

The two methods should be taught with equal importance to todays budding engineers. Without proper knowledge of both, it will be as if the blind are leading the blind. The fundamental beliefs and laws for all engineers dictate that health and well-being of the general public must be held above all else. Why then, should engineers not adopt a new standard that may be able to save thousands of lives and maybe more?

Appendix A

Appendix

Answer Locator

1. The title is located at the top of the first page.
2. The differences in deterministic and probabilisitic design methodologies are located in ”Limits of the Palmgren-Miner’s Method” and ”Reasons to Consider the Reliability Method”. The definitions of the two are located in ”Analyzing the Traditional Method of Engineering Design” and ”Safety and Reliability”.
3. Distinct applications are discussed in ”Uses of Probabilistic Design”.
4. The Analysis of a probabilistic problem is found in ”Reliability Method Application”.

Bibliography

[1] Bogdanovich, Sergei P., and Alexander E. Bogdanovich. "Analytical Probabilistic Modeling of Initial Failure and Reliability of Laminated Composite Structures." International Journal of Solids and Structures 35.7 (1998): 665-85. Print.

[2] Haldar, Achintya, and Sankaran Mahadevan. Basic Concept of Reliability. Probability, Reliability, and Statistical Methods in Engineering Design. Ed. Wayne Anderson, New York: John Wiley and Sons, 2000. 1-8.

[3] Kamiski, M. "On Probabilistic Fatigue Models for Composite Materials." International Journal of Fatigue 24.2-4 (2002): 477-95. Print.

[4] Kawai, M., S. Yajima, A. Hachinohe, and Y. Takano. "Off-Axis Fatigue Behavior of Unidirectional Carbon Fiber-Reinforced Composites at Room and High Temperatures." Journal of Composite Materials 35.7 (2001): 545-76. Print.

[5] Liu, Y., and S. Mahadevan. "Probabilistic Fatigue Life Prediction of Multidirectional Composite Laminates." Composite Structures 69.1 (2005): 11-19. Print.

[6] O'Shea, Michael J. "Snowboard Jumping, Newtons Second Law and the Force on Landing." Physics Education 39.4 (2004): 335-41. Print.

[7] Onkar, A., C. Upadhyay, and D. Yadav. "Probabilistic Failure of Laminated Composite Plates Using the Stochastic Finite Element Method." Composite Structures 77.1 (2007): 79-91. Print.

[8] Rognin, F., F. Abdi, V. Kunc, M. Lee, and K. Nikbin. "Probabilistic Methods in Predicting Damage under Multi-stage Fatigue of Composites Using Load Block Sequences." Procedia Engineering 1.1 (2009): 55-58. Print.